Student Number:									

Sydney Girls High School
2023
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks:

100

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8–37)

- Attempt Questions 11–40
- Allow about 2 hours and 45 minutes for this section

THIS IS A TRIAL PAPER ONLY
It does not necessarily reflect the format or the content of the 2023 HSC Examination Paper in this subject.

Question	M.C	11-23	24-30	31-40	
Total	/10	/31	/23	/36	/100

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet
Trial HSC Mathematics Advanced

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 = ?$ **(A) 2 (B) 6 (C) 8 (D) 9**

A ○ **B** ● **C** ○ **D** ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● **B** ✕ **C** ○ **D** ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ✕ **B** ● **C** ○ **D** ○

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1) Which is an example of discrete numerical data?

A. The weight of baby goats

B. The month in which birthday falls

C. The number of biscuits in a packet

D. A grade of A to D in an exam

2) For the random variable, X, it is known that $E(X) = 3.4$. If $E(X^2) = 13.6$, which of these is closest to the standard deviation σ ?

A. 1.84

B. 11.56

C. 1.43

D. 2.04

3) It is given that $I = \int_1^2 \ln x \, dx = 2\ln 2 - 1$. The approximation of I using the Trapezoidal rule with 3 function values is:

A. Smaller by 2.6%
B. Larger by 2.6%
C. Smaller by 97.4%
D. Larger by 97.4%

4) Given the point $P(\pi, 2)$ lies on $y = f(x)$ and $f'(\pi) = 0$ and $f''(\pi) = -e^2$, which statement is true about P?

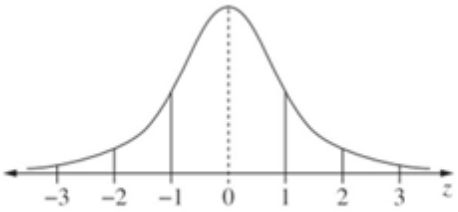
A. P is point of inflexion

B. P is a maximum turning point

C. P is a minimum turning point

D. P is horizontal point of inflexion

5) The graph below represents a standard normal distribution.



If the mean is 34 and standard deviation is 2, which of the following statements about this curve is false?

A. 5% of the scores are less than 30

B. The mode is 34

C. The median is 34

D. 68% of the scores are between 32 and 36

6) There are 24 girls and 16 boys in a class. From the class, $\frac{3}{8}$ of the girls and $\frac{1}{4}$ of the boys walk to school. One student who walks to school is chosen at random. Find the probability that the student is a boy.

A. $\frac{4}{13}$

B. $\frac{9}{64}$

C. $\frac{3}{32}$

D. $\frac{2}{5}$

7) $\frac{d}{dx}\log_e\left(\frac{9-25x^2}{5x+3}\right)$ is equivalent to which statement?

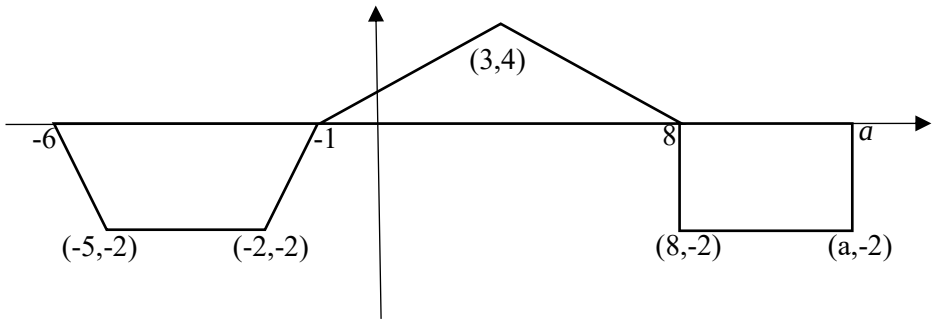
A. $\frac{-5}{3+5x}$

B. $\frac{5(3-5x)}{5x-3}$

C. $\frac{-5}{3-5x}$

D. $\frac{-5(3-5x)}{5x+3}$

8) For the graph of the function shown below, it is given that $\int_{-4}^{a} f(x)dx = 0$. Find the value of a.



A. 13
B. 14.5
C. 12.5
D. 12

9) A water tank holds 1250 litres of water. Water is flowing out of the tank at the rate R litres per minute where $R = 100t$ after t minutes. How long does it take the tank to empty, in minutes?

A. 15

B. 5

C. 25

D. 10

10) If $\tan^2\theta = 1 - e^2$, then the value of $\sec\theta + \tan^3\theta \operatorname{cosec}\theta$ is equal to:

A. $\left(2+e^2\right)^{\frac{3}{2}}$

B. $\left(2-e^2\right)^{\frac{3}{2}}$

C. $\left(2+e^2\right)^{\frac{1}{2}}$

D. $\left(2-e^2\right)^{\frac{1}{2}}$

Mathematics Advanced

Section II Answer Booklet 1

Section II

90 marks

Attempt Questions 11–40

Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 30 (54 marks)

Booklet 2 – Attempt Questions 31 – 40 (36 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Please clearly mark the questions written on the papers provided at the end of the paper.

Question 11 (2 marks)

Find the sum of the series 5 + 11 + 17 + ……+ 95. (2)

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Question 12 (2 marks)

Solve: $\sin 3x - \frac{1}{2} = 0$, where $0 \le x \le \pi$ (2)

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Question 13 (3 marks)

Two points have coordinates $P(4\ ,\ b^2 + 2b + 2)$ and $Q(3\ ,\ 3b + 3)$. It is given that the line PQ is parallel to the line $x + y = 1$. Find the value(s) of b. (3)

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Question 14 (2 marks)

Let $y = \cos^3 2x$. Find $\frac{dy}{dx}$. (2)

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Question 15 (2 marks)

The weekly salaries of workers at a factory are normally distributed with a mean of 410 dollars and a standard deviation of 40 dollars. A worker is chosen at random. Using the empirical rule, determine the probability of choosing a worker with a salary between 330 and 370 dollars. (2)

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Question 16 (2 marks)

Sketch the curve $y = |x-5|+2$, showing all important features. (2)

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Question 17 (3 marks)

Given $f(x) = \dfrac{x}{2-x}$:

i) Write down the domain in set notation. (1)

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ii) Show that the function $f(x)$ is increasing throughout the domain. (2)

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Question 18 (1 mark)

Find: $\int_{-\sqrt{5}}^{\sqrt{5}} \sqrt{5-x^2}\ dx$ (1)

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Question 19 (2 marks)

Prove the following identity: (2)

$$\sqrt{\frac{1-\sin x}{1+\sin x}} = \sec x - \tan x$$

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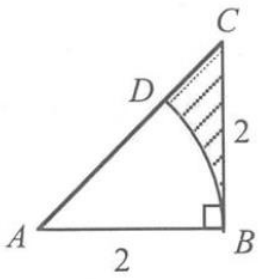
Question 20 (3 marks)

ABC is a triangle, right-angled at B. BD is an arc of a circle with centre A. $AB = BC = 2\,\text{cm}$. Find the exact area of the shaded region. (3)



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Question 21 (3 marks)

For the events A and B from a sample space, $P(A|B) = \frac{1}{4}$ and $P(B|A) = \frac{1}{3}$.
Let $P(A \cap B) = m$.

i) Find $P(B)$ in terms of m. (1)

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ii) Find $P(B' \cap A')$ in terms of m. (2)

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Question 22 (1 mark)

A cake is removed from the oven at 2:00 pm. Between 2:00 pm and 2:20 pm the temperature of the cake has fallen and the rate of change in temperature has also decreased. Draw a graph of temperature as a function of time that best fits this description. (1)

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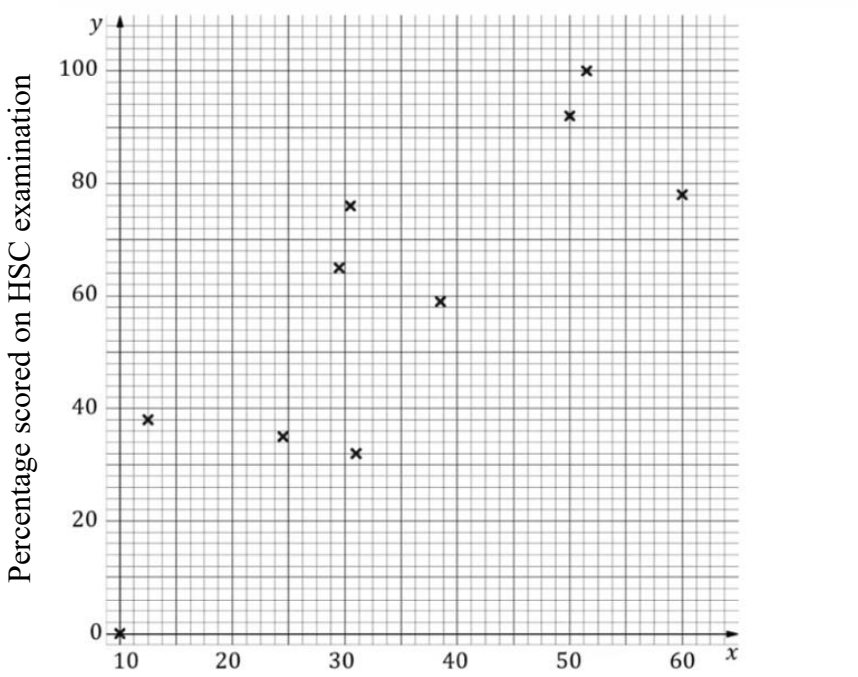
Question 23 (5 marks)

In the lead up to their 2023 Mathematics Advanced HSC examination, students revised by completing past trial examinations for this subject.
The scatter diagram below shows the number of past trial examinations completed by a group of students enrolled in Mathematics Advanced, and the percentage score they received in their HSC examination.



Number of past trial examinations completed

The equation of the regression line is $y = 18 + 1.3x$.

i) Draw the regression line onto the scatter diagram above. (2)

ii) Interpret the meaning of the values 18 and 1.3 in the context given above. (2)

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iii) Explain why the regression line given should not be used to estimate the percentage when someone has completed 80 past trial examinations. (1)

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Question 24 (3 marks)

A bag contains 5 white balls and 3 black balls. Two balls are selected at random from the bag, without replacement. Let X be the number of black balls drawn.

i) Construct the probability distribution table. (2)

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ii) Find the expected value of X. (1)

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Question 25 (2 marks)

Evaluate: $\int_1^2 (3x+2)^{-1}\, dx$ (Leave your answer in exact form.) (2)

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Question 26 (2 marks)

Find: $\int \frac{3x}{e^{-x^2+6}}\, dx$ (2)

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Question 27 (6 marks)

For the function $f(x) = x^3 - 3x + 2$:

i) Find the stationary points and determine their nature. (3)

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ii) Find the point of inflexion. (2)

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iii) Using parts i) and ii), sketch the graph of $y = f(x)$. (1)

Question 28 (2 marks)

The life span of a particular species of sea turtle is normally distributed with a mean of 170 years and a standard deviation of 35 years. By using the table or otherwise, find the probability that a random chosen turtle will live for more than 220 years, correct to 4 decimal places. (2)

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z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706

Question 29 (4 marks)

The continuous random variable *X* has probability density function *f(x)* where

$$f(x) = \begin{cases} ke^{\frac{-x}{2}}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

i) Show that $k = \frac{1}{2}$. (2)

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ii) What is the probability that the random variable *X* has a value that lies between $\frac{1}{3}$ and $\frac{1}{2}$? Leave your answer in terms of *e*. (2)

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Question 30 (4 marks)

A particle moves in a straight line so that its displacement, x metres, at time t seconds is given by:

$$x = 6t + 3\ln(3t^2 + 2)$$

i) Find the initial displacement. Leave your answer in exact form. (1)

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ii) Prove that the particle is never at rest. (2)

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iii) Hence, find the limiting velocity. (1)

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Section II extra writing space

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Student Number:									

Sydney Girls High School
2023

TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

Mathematics Advanced

Section II Answer Booklet 2

Attempt Questions 31 – 40 (36 marks)

Instructions:

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces do not provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the question booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 31 (5 marks)

Consider the function $y = (x-1)\log_e 2x$.

i) Find $\frac{dy}{dx}$. (2)

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ii) Hence show that : $\int_{\frac{1}{2}}^{1} \log_e 2x \, dx = \log_e 2 - \frac{1}{2}$ (3)

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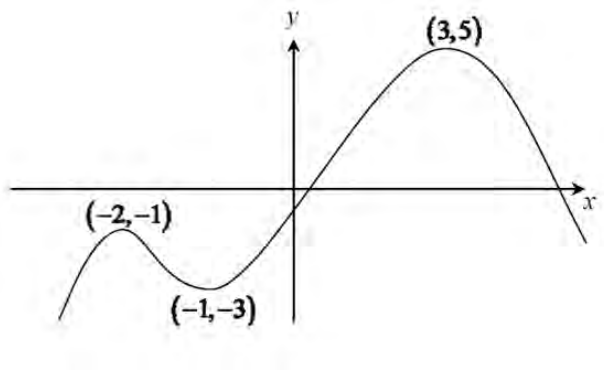
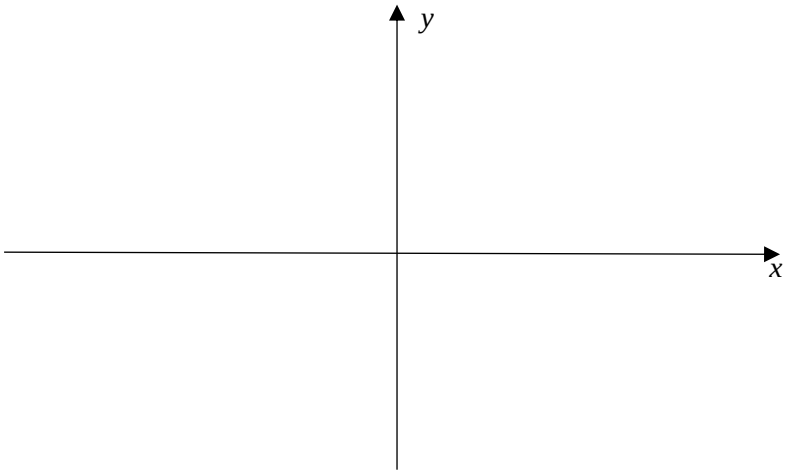
Question 32 (4 marks)



The diagram shows the graph of the function $y = f(x)$, with the coordinates of the turning points shown.

i) On the number plane shown below, sketch the graph of $y = f'(x)$. (2)



ii) Find the area bounded by the graph of $y = f'(x)$ and the x-axis, without finding the equation of the curve. (2)

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Question 33 (4 marks)

The sum of the first three terms of a geometric series is 152 and the sum of the next three terms is 513. Find:

i) the common ratio (3)

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ii) the first term (1)

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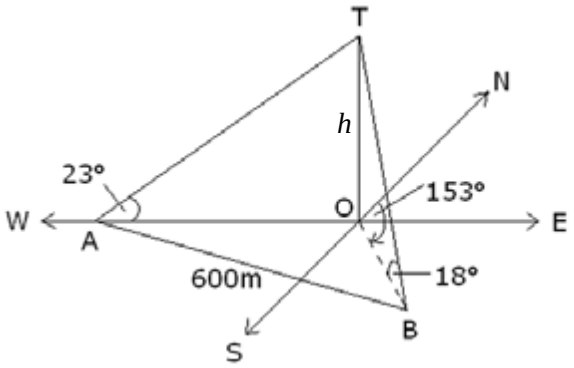
Question 34 (4 marks)

The angle of elevation to the top of a building from a point due west of the building is $23°$. Point *B* has a bearing of $153°$ T from the base of the building and the angle of elevation to the top of the building is $18°$. If *A* and *B* are 600 m apart:



i) Show that $\angle AOB = 117°$. (1)

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ii) Evaluate the height of the building *h*, correct to the nearest metre. (3)

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Question 35 (3 marks)

Find the exact value of x given $\log_3 x = \log_x \sqrt[9]{3}$. (3)

Question 36 (4 marks)

Donald is playing a game with two ordinary dice such that he wins if the sum of the two numbers on the dice is 6. If the sum is 5 then he loses. If the sum is anything else it is a draw and the player plays again until he wins or loses.

i) Find the probability that Donald wins on the first throw. (1)

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ii) What is the probability that Donald wins on the first, second or third throw? (1)

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iii) Calculate the probability that Donald eventually wins the game. (2)

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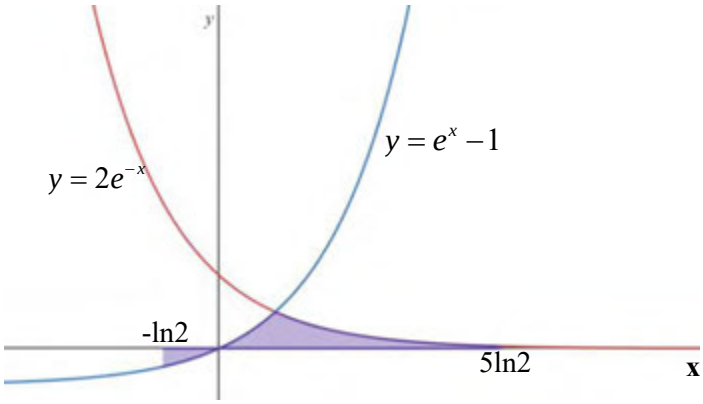
Question 37 (3 marks)

The diagram below shows the graph of $y = 2e^{-x}$ and $y = e^{x} - 1$.



Find the exact area of the shaded region between $x = -\ln 2$ and $x = 5\ln 2$. (3)

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Question 38 (1 mark)

Find: $\int \frac{1}{x \ln x}\, dx$ (1)

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Question 39 (3 marks)

If $y^2 = (2m - y)(2n - y)$, show that $\frac{1}{m}, \frac{1}{y}, \frac{1}{n}$ is an arithmetic sequence. (3)

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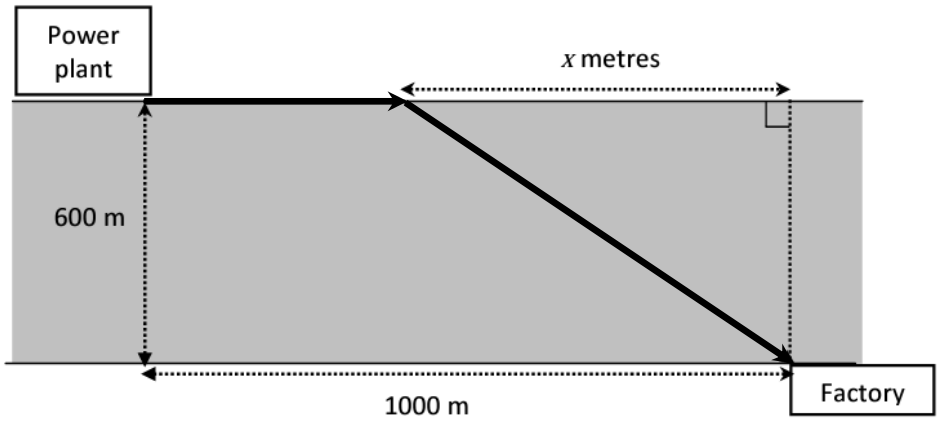
Question 40 (5 marks)

An underground power line is to run from a power plant at one side of a river to a factory at the other side, 1000 metres downstream. The river is 600 m wide and has straight banks. The diagram below shows the proposed route of the power line. It follows the river bank for a distance before crossing the river to the factory. The cost of laying the power line under land is \$140 per metre, and the cost of laying the power line underwater is \$180 per metre. It is required to find the route costing the least to install the power line.



i) Find the total cost of the line in terms of x. (2)

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ii) Find the route that gives the minimum cost. (3)

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Section II extra writing space

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Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

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1) Which is an example of discrete numerical data?

A. The weight of baby goats

B. The month in which birthday falls

(C.) The number of biscuits in a packet

D. A grade of A to D in an exam

2) For the random variable, X, it is known that $E(X) = 3.4$. If $E(X^2) = 13.6$, which of these is closest to the standard deviation σ?

A. 1.84

B. 11.56

(C.) 1.43

D. 2.04

3) It is given that $I = \int_1^2 \ln x \, dx = 2\ln 2 - 1$. The approximation of I using the Trapezoidal rule with 3 function values is:

(A.) Smaller by 2.6%

B. Larger by 2.6%

C. Smaller by 97.4%

D. Larger by 97.4%

4) Given the point $P(\pi, 2)$ lies on $y = f(x)$ and $f'(\pi) = 0$ and $f''(\pi) = -e^2$, which statement is true about P?

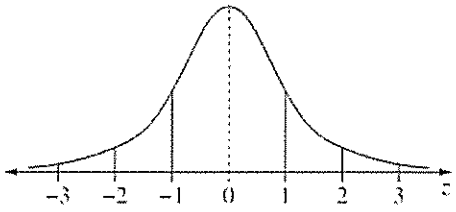
A. P is point of inflexion

B. P is a maximum turning point

C. P is a minimum turning point

D. P is horizontal point of inflexion

5) The graph below represents a standard normal distribution.



If the mean is 34 and standard deviation is 2, which of the following statements about this curve is false?

A. 5% of the scores are less than 30

B. The mode is 34

C. The median is 34

D. 68% of the scores are between 32 and 36

6) There are 24 girls and 16 boys in a class. From the class, $\frac{3}{8}$ of the girls and $\frac{1}{4}$ of the boys walk to school. One student who walks to school is chosen at random. Find the probability that the student is a boy.

A. $\frac{4}{13}$

B. $\frac{9}{64}$

C. $\frac{3}{32}$

D. $\frac{2}{5}$

7) $\frac{d}{dx}\log_e\left(\frac{9-25x^2}{5x+3}\right)$ is equivalent to which statement?

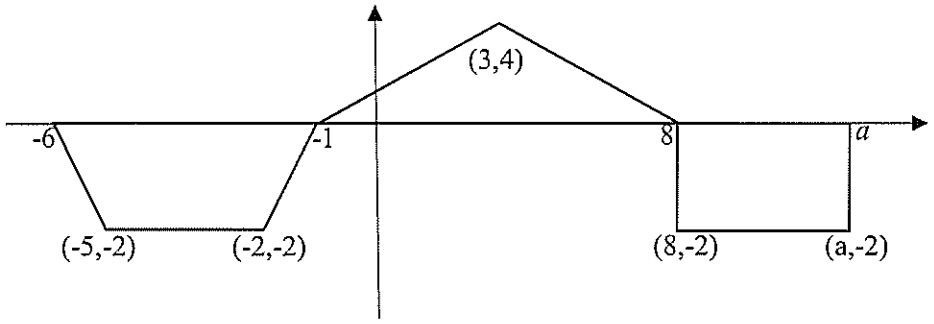
A. $\frac{-5}{3+5x}$

B. $\frac{5(3-5x)}{5x-3}$

C. $\frac{-5}{3-5x}$

D. $\frac{-5(3-5x)}{5x+3}$

8) For the graph of the function shown below, it is given that $\int_{-4}^{a} f(x)dx = 0$. Find the value of a.



A. 13
B. 14.5
C. 12.5
D. 12

9) A water tank holds 1250 litres of water. Water is flowing out of the tank at the rate R litres per minute where $R = 100t$ after t minutes. How long does it take the tank to empty, in minutes?

A. 15
B. 5
C. 25
D. 10

10) If $\tan^2\theta = 1 - e^2$, then the value of $\sec\theta + \tan^3\theta \operatorname{cosec}\theta$ is equal to:

A. $\left(2+e^2\right)^{\frac{3}{2}}$

B. $\left(2-e^2\right)^{\frac{3}{2}}$

C. $\left(2+e^2\right)^{\frac{1}{2}}$

D. $\left(2-e^2\right)^{\frac{1}{2}}$

Question 11 (2 marks)

Find the sum of the series $5 + 11 + 17 + \ldots\ldots + 95$. (2)

A.P $a = 5$ $d = 6$ $l = 95$ $n = ?$ $S_n = ?$

$95 = 5 + (n-1)6$

$90 = 6(n-1)$

$6n = 96$

$n = 16$ ✓

$S_n = \frac{n}{2}(a+l)$

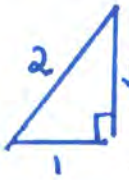
$= \frac{16}{2}(5+95)$

$= 8(100)$

$= 800$ ✓

Question 12 (2 marks)

Solve: $\sin 3x - \frac{1}{2} = 0$, where $0 \le x \le \pi$ (2)



$\sin 3x = \frac{1}{2}$ ✓✓ $0 \le 3x \le 3\pi$

$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{6} + 2\pi, \frac{5\pi}{6} + 2\pi$ ✓

$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$ ✓

Many students were missing solns.

Question 13 (3 marks)

Two points have coordinates $P(4\ ,\ b^2+2b+2)$ and $Q(3\ ,\ 3b+3)$. It is given that the line PQ is parallel to the line $x+y=1$. Find the value(s) of b. (3)

$y = -x + 1$

$m = -1$

$\frac{y_2 - y_1}{x_2 - x_1} = -1$ ✓

$\frac{3b+3-(b^2+2b+2)}{3-4} = -1$ ✓

$3b + 3 - b^2 - 2b - 2 = 1$

$-b^2 + b + 1 = 1$

$-b^2 + b = 0$

$b^2 - b = 0$

$b(b-1) = 0$

$b = 0, 1$ ✓

Question 14 (2 marks)

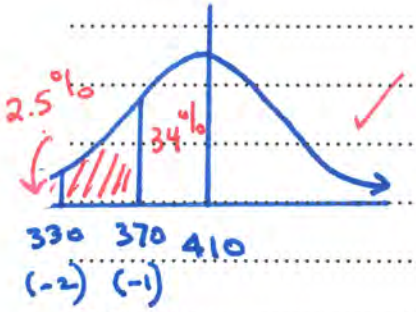
Let $y = \cos^3 2x$. Find $\frac{dy}{dx}$. (2)

$\frac{dy}{dx} = 3(\cos 2x)^2 \times -2\sin 2x$ ✓

$= -6 \sin 2x \cos^2 2x$ ✓

Question 15 (2 marks)

The weekly salaries of workers at a factory are normally distributed with a mean of 410 dollars and a standard deviation of 40 dollars. A worker is chosen at random. Using the empirical rule, determine the probability of choosing a worker with a salary between 330 and 370 dollars. (2)

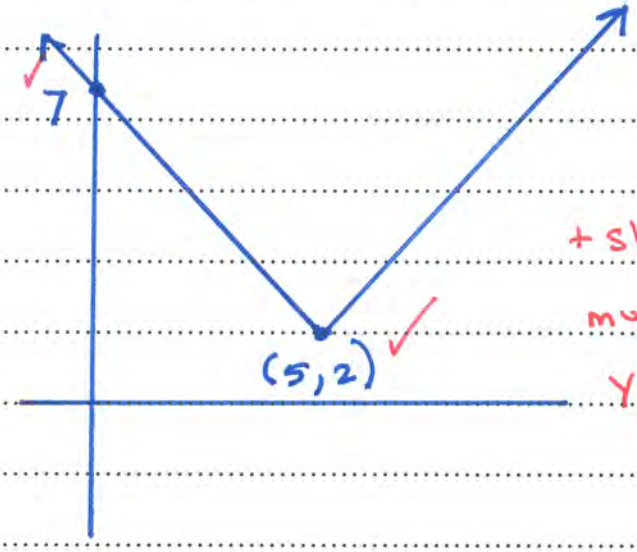


$P(330 < W < 370) = 50 - 34 - 2.5$ ✓

$= 13.5\%$ ✓

Question 16 (2 marks)

Sketch the curve $y = |x-5|+2$, showing all important features. (2)



+ shape also important.
must use a ruler for your graph.

Question 17 (3 marks)

Did not accept $x \neq 2$

Given $f(x) = \dfrac{x}{2-x}$:

i) Write down the domain in set notation. (1)

$(-\infty, 2) \cup (2, \infty)$ Also accepted $x \in \mathbb{R}, x \neq 2$

Some students used $\wedge$ this is not correct.

All real, $x \neq 2$ ✓

ii) Show that the function $f(x)$ is increasing throughout the domain. (2)

$$f'(x) = \frac{(2-x)\cdot 1 - x(-1)}{(2-x)^2}$$

$$= \frac{2-x+x}{(2-x)^2}$$

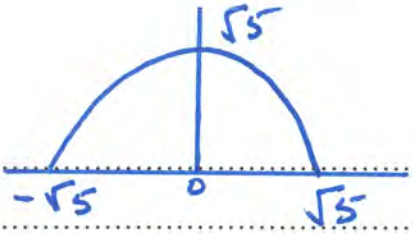
$$= \frac{2}{(2-x)^2}$$ ✓

> 0 for all x ✓

$\therefore f(x)$ is increasing throughout the domain. ✓

Question 18 (1 mark)

Find: $\displaystyle\int_{-\sqrt{5}}^{\sqrt{5}} \sqrt{5-x^2}\, dx$ (1)



* can not answer this question using integration.

$= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 5$

$= \frac{5\pi}{2}$ ✓

No units required as it is an integral. Not asking for area.

Question 19 (2 marks)

Prove the following identity: (2)

$$\sqrt{\frac{1-\sin x}{1+\sin x}} = \sec x - \tan x$$

L.H.S $= \sqrt{\frac{1-\sin x}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}}$

$= \sqrt{\frac{(1-\sin x)^2}{1-\sin^2 x}}$

$= \sqrt{\frac{(1-\sin x)^2}{\cos^2 x}}$

$= \frac{(1-\sin x)}{\cos x}$ ✓

R.H.S $= \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

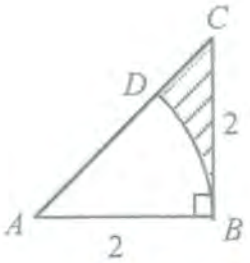
$= \frac{1-\sin x}{\cos x}$ ✓

L.H.S = R.H.S

$\therefore \sqrt{\frac{1-\sin x}{1+\sin x}} = \sec x - \tan x$

Question 20 (3 marks)

ABC is a triangle, right-angled at B. BD is an arc of a circle with centre A. $AB = BC = 2$ cm.
Find the exact area of the shaded region. (3)



$A_T = \frac{1}{2} \times 2 \times 2 = 2$ ✓

$A_S = \frac{\theta}{360} \pi r^2$

$= \frac{45}{360} \pi (4)$

$= \frac{\pi}{2}$ ✓

$\neq \frac{3\pi}{2}$ (be careful!!)

$\therefore$ shaded area

$= 2 - \frac{\pi}{2}$

$= \frac{4-\pi}{2}$ u^2 ✓

Question 21 (3 marks)

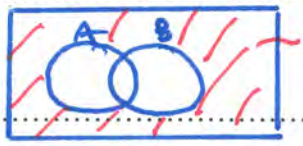
For the events A and B from a sample space, $P(A|B) = \frac{1}{4}$ and $P(B|A) = \frac{1}{3}$.
Let $P(A \cap B) = m$.

i) Find $P(B)$ in terms of m. (1)

$\frac{1}{4} = \frac{P(A \cap B)}{P(B)}$ $\therefore P(B) = 4m$ ✓

$\frac{1}{4} = \frac{m}{P(B)}$

ii) Find $P(B' \cap A')$ in terms of m. (2)



$(B' \cap A')$

$P(B' \cap A') = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [3m + 4m - m]$

$= 1 - 6m$

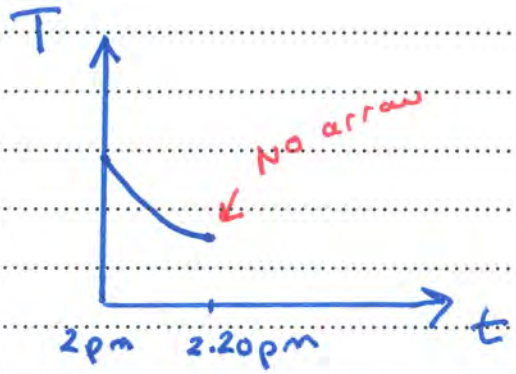
A lot of students did not apply the addition rule correctly.

$\frac{1}{3} = \frac{P(A \cap B)}{P(A)}$ ✓

$P(A) = 3m$

Question 22 (1 mark)

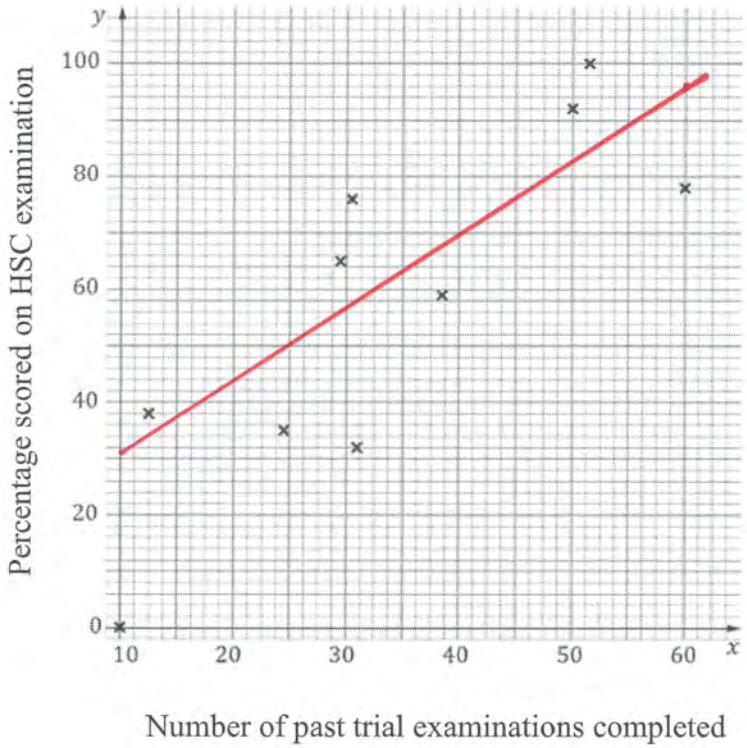
A cake is removed from the oven at 2:00 pm. Between 2:00 pm and 2:20 pm the temperature of the cake has fallen and the rate of change in temperature has also decreased. Draw a graph of temperature as a function of time that best fits this description. (1)



If your graph stopped at 2.20pm you were awarded the mark (although there should not be an arrow)

Question 23 (5 marks)

In the lead up to the 2023 Mathematics Advanced HSC examination, students revised by completing past trial examinations for Mathematics Advanced.
The scatter diagram below shows the number of past trial examinations completed by a group of students, and the percentage score they received in their HSC examination.



Students needed to recognise the x axis started at 10 not 0. Therefore the y-int is 31 not 18.
To draw the regression line you needed two points using the regression line.
Some students confused the regression line with a line of best fit and lost the 2 marks.

The equation of the regression line is $y = 18 + 1.3x$.

i) Draw the regression line onto the scatter diagram. (2)

$x = 10$ $y = 31$; $x = 60$ $y = 96$.

ii) Interpret the meaning of the values 18 and 1.3 in the context given above. (2)

18 is the estimated mark if 0 past HSC exams were completed. 1.3 is the predicted % increase in mark for every exam completed.

iii) Explain why the regression line given should not be used to estimate the percentage when someone has completed 80 past trial examinations. (1)

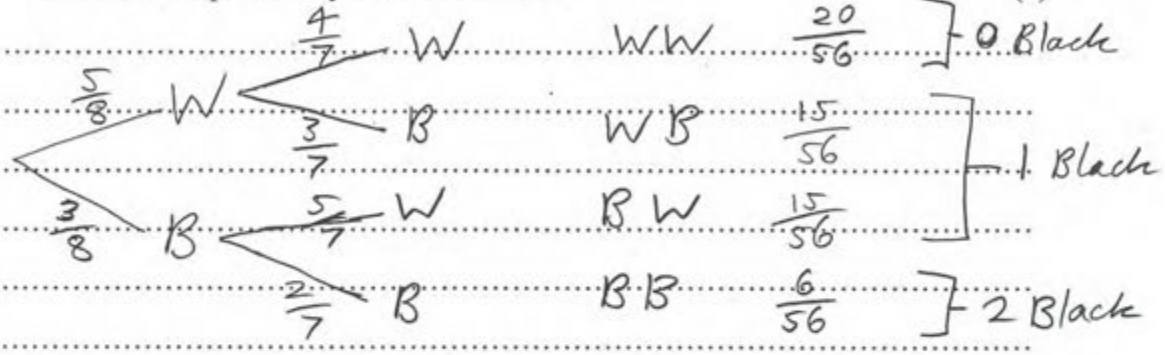
If the regression line is used the predicted mark for completing 80 past HSC exams is 122%, which is impossible. Extrapolation in this case is therefore not viable.

Answers should mention the 122% and extrapolation. Some responses did not make sense and were ∴ not awarded the mark.

Question 24 (3 marks)

A bag contains 5 white balls and 3 black balls. Two balls are selected at random from the bag, without replacement. Let X be the number of black balls drawn.

i) Construct the probability distribution table. (2)



X	0	1	2
$P(X)$	$\frac{20}{56}$	$\frac{30}{56}$	$\frac{6}{56}$
	$=\frac{5}{14}$	$=\frac{15}{28}$	$=\frac{3}{28}$

Comment: the outcome of 0 (zero) black balls needs to be included in the table.

ii) Find the expected value of X. (1)

$$E(X) = \sum X \cdot P(X)$$

$$= 0 \times \frac{5}{14} + 1 \times \frac{15}{28} + 2 \times \frac{3}{28}$$

$$= \frac{3}{4}$$

Question 25 (2 marks)

Evaluate: $\int_1^2 (3x+2)^{-1}\,dx$ (Leave your answer in exact form.) (2)

$= \int \frac{1}{3} \times \frac{3}{3x+2} \cdot dx$

$= \frac{1}{3}\left[\ln|3x+2|\right]_1^2$

$= \frac{1}{3}(\ln 8 - \ln 5)$

$= \frac{1}{3}\ln\left(\frac{8}{5}\right)$

$= \int 3x(e^{-x^2+6})^{-1}\,dx$

Question 26 (2 marks)

Find: $\int \frac{3x}{e^{-x^2+6}}\,dx$ (2)

$= 3\int x \cdot e^{x^2-6}\,dx$

$= \frac{3}{2}\int 2x \cdot e^{x^2-6}\,dx$

$= \frac{3}{2}e^{x^2-6} + c$

Question 27 (6 marks)

For the function $f(x) = x^3 - 3x + 2$:

i) Find the stationary points and determine their nature. (3)

$f(x) = x^3 - 3x + 2$

$f'(x) = 3x^2 - 3 \qquad f''(x) = 6x$

Stat. pts when $f'(x) = 0$

$3x^2 - 3 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

When $x = 1$: $f''(x) = 6 > 0$ ∪ ∴ Min. stat. pt. at $(1, 0)$

When $x = -1$: $f''(x) = -6 < 0$ ∩ ∴ Max. stat. pt. at $(-1, 4)$

Comment: Points include (x, y) not just an x value. Test must also be included.

ii) Find the point of inflexion. (2)

Possible Inflection pt. when $f''(x) = 0$

$6x = 0$

$x = 0$

Test:

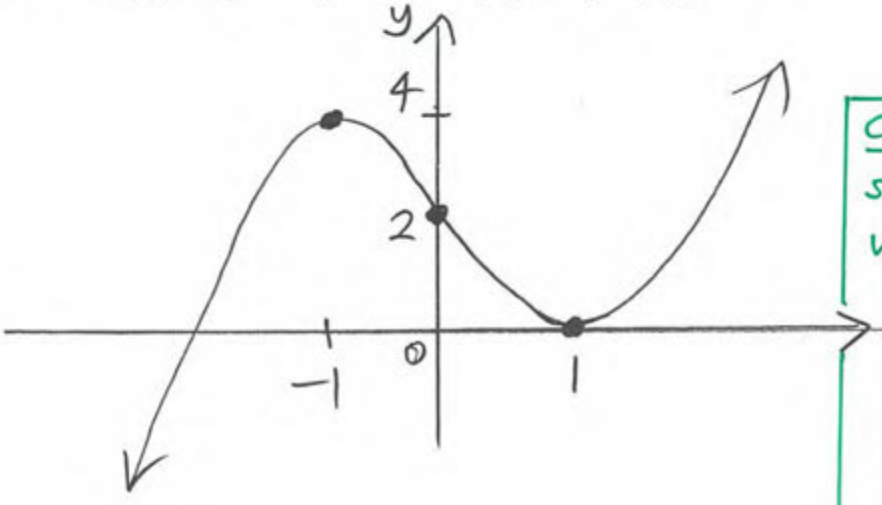
x	-1	0	1
$f''(x)$	-6	0	6
	<		>

Concavity changes

∴ Inflexion pt. at $(0, 2)$

Comment: Test must be included to show concavity change with values, not just + or −

iii) Using parts i) and ii), sketch the graph of $y = f(x)$. (1)



Comment: Graphs should be drawn with a smooth curve to correctly show concavity and inflection pt. Draw graphs larger to clearly label points.

Question 28 (2 marks)

The life span of a particular species of sea turtle is normally distributed with a mean of 170 years and a standard deviation of 35 years. By using the table or otherwise, find the probability that a random chosen turtle will live for more than 220 years, correct to 4 decimal places. (2)

$\mu = 170 \quad \sigma = 35 \quad z = \frac{220 - 170}{35}$

$\doteqdot 1.43$

$P(X > 220) = 1 - P(X \leq 220)$

$= 1 - 0.9236$

$= 0.0764$

Comment: If using the table below, you need to round the "z" score to 2 decimal places correctly.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706

Question 29 (4 marks)

The continuous random variable X has probability density function $f(x)$ where

$$f(x)=\begin{cases} ke^{\frac{-x}{2}} & x\geq 0 \\ 0 & elsewhere \end{cases}$$

i) Show that $k=\frac{1}{2}$. (2)

For PDF: $\int_0^\infty ke^{-\frac{x}{2}}dx=1$

$k\int_0^\infty e^{-\frac{x}{2}}dx=1$

$k\left[\frac{e^{-\frac{x}{2}}}{-\frac{1}{2}}\right]_0^\infty=1$

$k\left[-2e^{-\frac{x}{2}}\right]_0^\infty=1$

$k\left(-\frac{2}{e^\infty}+\frac{2}{e^0}\right)=1$

$k(0+2)=1$

$2k=1$

$k=\frac{1}{2}$

ii) What is the probability that the random variable X has a value that lies between $\frac{1}{3}$ and $\frac{1}{2}$? Leave your answer in terms of e. (2)

$\int_{\frac{1}{3}}^{\frac{1}{2}}\frac{1}{2}e^{-\frac{x}{2}}dx=\left[\frac{1}{2}x-2e^{-\frac{x}{2}}\right]_{\frac{1}{3}}^{\frac{1}{2}}$

$=\left[-e^{-\frac{x}{2}}\right]_{\frac{1}{3}}^{\frac{1}{2}}$

$=-e^{-\frac{1}{4}}+e^{-\frac{1}{6}}$

Question 30 (4 marks)

A particle moves in a straight line so that its displacement, x metres, at time t seconds is given by:

$$x = 6t + 3\ln(3t^2 + 2)$$

i) Find the initial displacement. Leave your answer in exact form. (1)

Initial when $t=0$:

$x = 0 + 3\ln 2$

$\therefore x = 3\ln 2$ metres

ii) Prove that the particle is never at rest. At rest when $v=0$ (2)

$x = 6t + 3\ln(3t^2+2)$

$v = 6 + 3\left(\frac{6t}{3t^2+2}\right)$

$\therefore v = 6 + \frac{18t}{3t^2+2}$

As $3t^2+2 > 0$ and $t \geq 0$,

$v > 0$

i.e. Velocity is always positive $\therefore$ never at rest

OR when $v=0$:

$-6 = \frac{18t}{3t^2+2}$

Not possible as $t \geq 0$

$\therefore v \neq 0 \therefore$ never at rest

OR $-6 = \frac{18t}{3t^2+2}$ (when $v=0$)

$18t^2 + 18t + 12 = 0$

$3t^2 + 3t + 2 = 0$

$\Delta = 9 - 4(3)(2)$

$= -15 < 0$

$\therefore$ No sol. $\therefore v \neq 0$

$\therefore$ never at rest

Comment: Various answers. Correct justification needed to be shown

iii) Hence, find the limiting velocity. (1)

As $t \to \infty$, $\frac{18t}{3t^2+2} \to 0$

$\therefore \lim_{t\to\infty}\left(6 + \frac{18t}{3t^2+2}\right) = 6$

$\therefore$ Limiting velocity $= 6$ m/s

Question 31 (5 marks)

Consider the function $y = (x-1)\log_e 2x$.

i) Find $\frac{dy}{dx}$. (2)

$\frac{dy}{dx} = \log_e 2x + \frac{2}{2x}(x-1)$

$= \log_e 2x + \frac{x-1}{x}$

$= \log_e 2x + 1 - \frac{1}{x}$

ii) Hence show that : $\int_{\frac{1}{2}}^{1} \log_e 2x \, dx = \log_e 2 - \frac{1}{2}$ (3)

$\int \log_e 2x + 1 - \frac{1}{x} = (x-1)\log_e 2x$

$\int \log_e 2x = (x-1)\log_e 2x - \int 1 - \frac{1}{x}$

$= \left[(x-1)\log_e 2x - x + \ln x\right]_{\frac{1}{2}}^{1}$

$= -1 - \left(-\frac{1}{2}\log_e 1 - \frac{1}{2} + \ln\frac{1}{2}\right)$

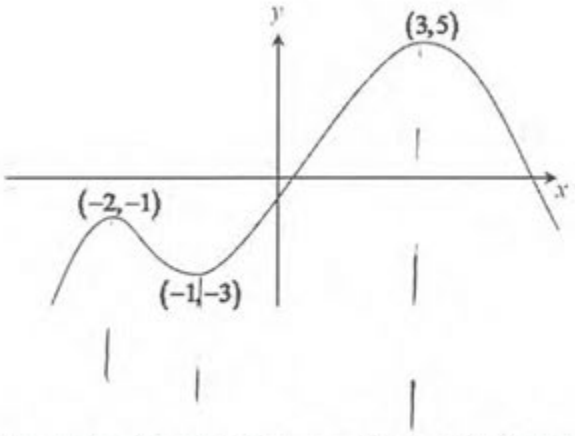
$= -1 + \frac{1}{2} - \ln\frac{1}{2}$

$= -\frac{1}{2} - \ln 2^{-1}$

$= -\frac{1}{2} + \ln 2$

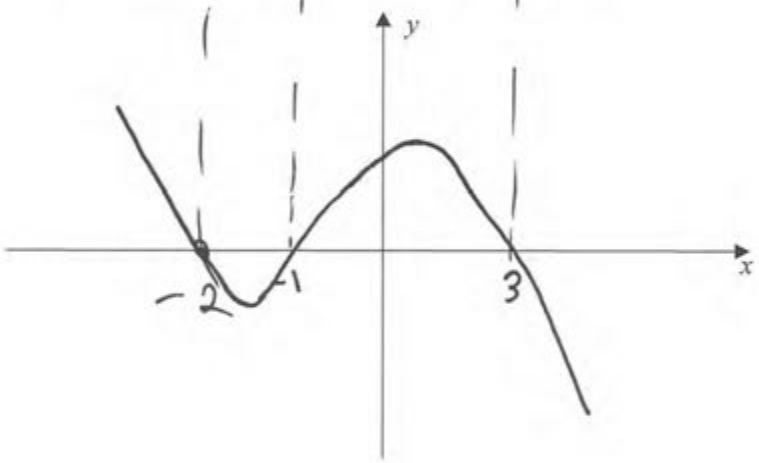
If the steps were not clearly shown, marks were deducted

Question 32 (4 marks)



The diagram shows the graph of the function $y = f(x)$, with the coordinates of the turning points shown.

i) On the number plane shown below, sketch the graph of $y = f'(x)$. (2)



ii) Find the area bounded by the graph of $y = f'(x)$ and the x-axis, without finding the equation of the curve. (2)

$$A = \left| \int_{-2}^{-1} f(x)\,dx \right| + \int_{-1}^{3} f(x)\,dx$$

$$= \left| F(-1) - F(-2) \right| + \left(F(3) - F(-1) \right)$$

$$= \left| -3 - (-1) \right| + \left(5 - -3 \right)$$

$$= |-2| + 8$$

$$= 10 \text{ u}^2$$

This question was done poorly

Question 33 (4 marks)

The sum of the first three terms of a geometric series is 152 and the sum of the next three terms is 513. Find:

i) the common ratio (3)

$a + ar + ar^2 = 152$

$ar^3 + ar^4 + ar^5 = 513$

$a(1 + r + r^2) = 152$

$ar^3(1 + r + r^2) = 513$

$ar^3\left(\frac{152}{a}\right) = 513$

$152r^3 = 513$

$r^3 = \frac{27}{8}$

$\therefore r = \frac{3}{2}$

If the students used this method it was much easier than using Sum formula

ii) the first term (1)

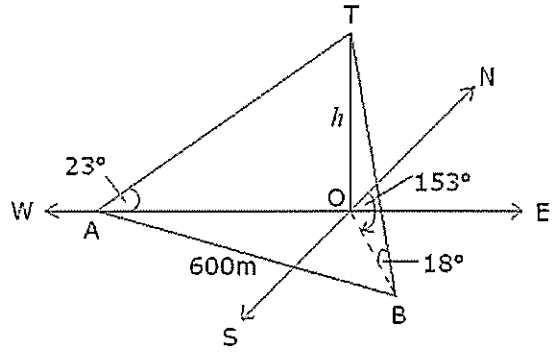
$a\left(1 + \frac{3}{2} + \frac{9}{4}\right) = 152$

$a = \frac{152}{\frac{19}{4}}$

$a = 32$

Question 34 (4 marks)

The angle of elevation to the top of a building from a point due west of the building is 23°. Point B has a bearing of 153° T from the base of the building and the angle of elevation to the top of the building is 18°. If A and B are 600 m apart:



i) Show that $\angle AOB = 117°$. (1)

$\angle AOB = 270° - 153°$

$= 117°$

ii) Evaluate the height of the building h, correct to the nearest metre. (3)

$\tan 23 = \frac{h}{OA}$ $\quad \tan 18° = \frac{h}{OB}$

$OA = \frac{h}{\tan 23}$ $\quad OB = \frac{h}{\tan 18}$

$600^2 = \left(\frac{h}{\tan 23}\right)^2 + \left(\frac{h}{\tan 18}\right)^2 - 2\left(\frac{h}{\tan 23}\right)\left(\frac{h}{\tan 18}\right)\cos 117°$

$600^2 = h^2\left(\frac{1}{\tan^2 23} + \frac{1}{\tan^2 18} - \frac{2\cos 117°}{\tan 23 \tan 18}\right)$

poorly done

$h^2 = \frac{600^2}{21.60555} = 16662.38$

$h = 129$ m

Question 35 (3 marks)

Find the exact value of x given $\log_3 x = \log_x \sqrt[9]{3}$. (3)

$$\text{Log}_3 x = \frac{\log_3 \sqrt[9]{3}}{\log_3 x}$$

$$(\log_3 x)^2 = \text{Log}_3 3^{\frac{1}{9}}$$

$$(\log_3 x)^2 = \frac{1}{9}$$

$$\text{Log}_3 x = \pm \frac{1}{3}$$

This question actually had two answers, but because of the 's' missing from 'values' I gave 2 marks for ± and 1 mark for the answer.

$$\text{Log}_3 x = \frac{1}{3} \qquad \text{Log}_3 x = -\frac{1}{3}$$

$$x = 3^{\frac{1}{3}} \qquad x = 3^{-\frac{1}{3}}$$

$$x = \sqrt[3]{3} \qquad = \frac{1}{\sqrt[3]{3}}$$

OR

$$\frac{\text{Log}_x x}{\log_x 3} = \log_x 3^{\frac{1}{9}} \qquad 9 = (\text{Log}_x 3)^2$$

$$\frac{1}{\log_x 3} = \frac{1}{9} \log_x 3 \qquad \log_x 3 = \pm 3$$

$$1 = \frac{1}{9} (\log_x 3)^2 \qquad x^3 = 3$$

$$x = \sqrt[3]{3}$$

$$x^{-3} = 3$$

$$x = 3^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{3}}$$

Question 36 (4 marks)

Donald is playing a game with two ordinary dice such that he wins if the sum of the two numbers on the dice is 6. If the sum is 5 then he loses. If the sum is anything else it is a draw and the player plays again until he wins or loses.

i) Find the probability that Donald wins on the first throw. (1)

$P(W) = \frac{5}{36}$

ii) What is the probability that Donald wins on the first, second or third throw? (1)

$P(L) = \frac{4}{36}$ $P(d) = \frac{27}{36}$

$P(W) + P(LW) + P(LLW) = \frac{5}{36} + \frac{5}{36} \times \frac{27}{36} + \frac{27}{36} \times \frac{27}{36} \times \frac{5}{36}$

$= \frac{185}{576}$

iii) Calculate the probability that Donald eventually wins the game. (2)

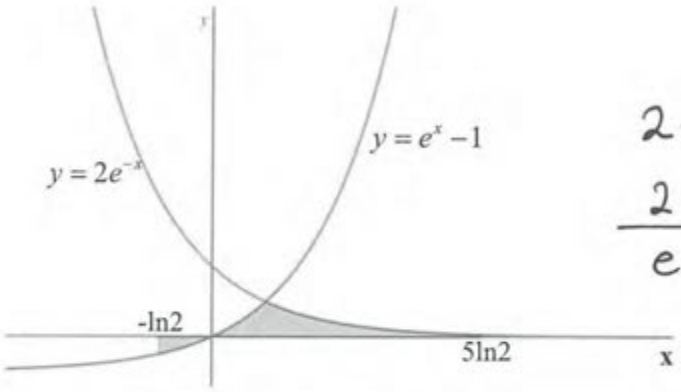
$a = \frac{5}{36}$ $\frac{5}{36}\left(1 + \frac{27}{36} + \left(\frac{27}{36}\right)^2 + \ldots\right)$

$r = \frac{27}{36}$ $S_\infty = \frac{5}{36}\left(\frac{1}{1 - \frac{27}{36}}\right)$

$= \frac{5}{9}$

Question 37 (3 marks)

The diagram below shows the graph of $y = 2e^{-x}$ and $y = e^{x} - 1$.



$2e^{-x} = e^{x} - 1$

$\frac{2}{e^{x}} = e^{x} - 1$

$2 = e^{2x} - e^{x}$

$e^{2x} - e^{x} - 2 = 0$

Let $e^{x} = m$

Find the exact area of the shaded region between $x = -\ln 2$ and $x = 5\ln 2$. (3)

$A = \int_{0}^{-\ln 2} e^{x} - 1 \, dx + \int_{0}^{\ln 2} e^{x} - 1 \, dx + \int_{\ln 2}^{5\ln 2} 2e^{-x}$

$m^{2} - m - 2 = 0$

$(m-2)(m+1) = 0$

$m = 2$ or $m = -1$

$e^{x} = 2$ $e^{x} \neq -1$

$x = \ln 2$

$A = \left[e^{x} - x\right]_{-\ln 2}^{0} + \left[e^{x} - x\right]_{0}^{\ln 2} + \left[-2e^{-x}\right]_{\ln 2}^{5\ln 2}$

$= e^{-\ln 2} + \ln 2 + e^{0} + e^{\ln 2} - \ln 2 - e^{0} - 2e^{-5\ln 2} + 2e^{-\ln 2}$

$= \frac{1}{2} - 1 + 2 - 1 - 2\left(2^{-5}\right) + 2\left(\frac{1}{2}\right)$

$= \frac{3}{2} - \frac{1}{16}$

$= \frac{23}{16}$

some students found this question hard.

Question 38 (1 mark)

Find: $\int \frac{1}{x \ln x}\, dx$ (1)

$= \int \frac{\frac{1}{x}}{\ln x}\, dx$ $\int \frac{f'(x)}{f(x)} = \ln(f(x)) + C$

$= \ln(\ln x) + C$ This was done poorly

Question 39 (3 marks)

If $y^2 = (2m - y)(2n - y)$, show that $\frac{1}{m}, \frac{1}{y}, \frac{1}{n}$ is an arithmetic sequence. (3)

This question was done poorly as well.

$y^2 = 4mn - 2my - 2ny + y^2$

$4mn - 2my - 2ny = 0$ $4mn = 2my + 2ny$

$4mn = y(2m + 2n)$

$\frac{1}{y} - \frac{1}{m} = \frac{1}{n} - \frac{1}{y}$ $y = \frac{4mn}{2m+2n}$

LHS

$\frac{1}{\frac{4mn}{2m+2n}} - \frac{1}{m}$ RHS $\frac{1}{n} - \frac{1}{\frac{4mn}{2m+n}}$

$= \frac{2m+2n}{4mn} - \frac{1}{m}$ $= \frac{1}{n} - \frac{2m+2n}{4mn}$

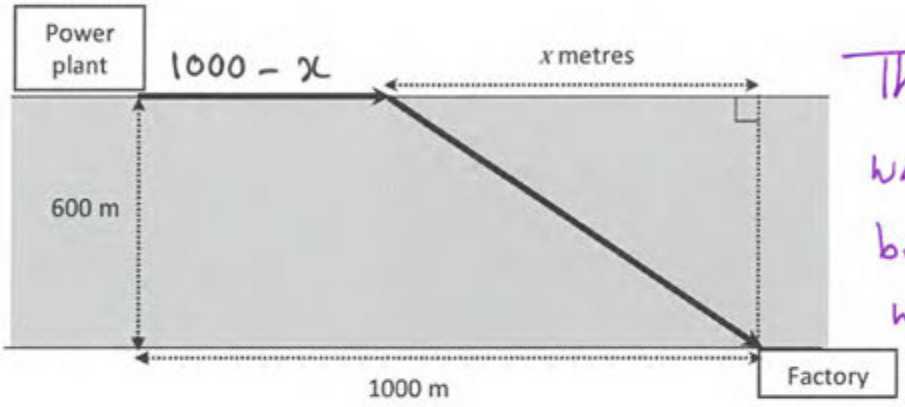
$= \frac{2m+2n-4n}{4mn}$ $= \frac{4m-2m-2n}{4mn}$

$= \frac{2m-2n}{4mn}$ $= \frac{2m-2n}{4mn} = \frac{m-n}{2mn}$

$= \frac{m-n}{2mn}$ LHS = RHS $\therefore$ A.P

Question 40 (5 marks)

An underground power line is to run from a power plant at one side of a river to a factory at the other side, 1000 metres downstream. The river is 600 m wide and has straight banks. The diagram below shows the proposed route of the power line. It follows the river bank for a distance before crossing the river to the factory. The cost of laying the power line under land is \$140 per metre, and the cost of laying the power line underwater is \$180 per metre. It is required to find the route costing the least to install the power line.



This question was done well but if Testing wasn't shown 1 mark was deducted

i) Find the total cost of the line in terms of x. (2)

$C = (1000 - x)(140) + \sqrt{(600^2 + x^2)}\,(180)$

ii) Find the route that gives the minimum cost. (3)

$C' = -140 + \frac{1}{2}(600^2 + x^2)^{-\frac{1}{2}} \cdot 2x \cdot 180$

$= -140 + \frac{180}{\sqrt{600^2 + x^2}}$

x	740	742.5	743
C'	-0.18	0	0.04

min

$C' = 0 \quad 140 = \frac{180x}{\sqrt{600^2 + x^2}}$

$14(360000 + x^2) = 18^2 x^2$

$196(x^2 + 360000) = 324x^2$

$x^2 = 551250$

$x = \sqrt{551250}$

$x = 742.32\ m$